Modelling the view factor of a 'grain-like' observer near a tilted pool fire via planar approximation approach

Christopher J. Budd*, Ugochukwu O. Ugwu†

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Abstract

Modelling the view factor, F, of a 'grain-like' observer near a tilted pool fire viewed to be cylindrical in shape, requires the observer (on the same ground level as the pool fire) to be as close as possible to the flame surface so that the flame surface is viewed as a plane. The orientation of the observer is to receive a maximum view, with the view factor integrated over the flame area 'seen' by the observer. The derived expression for F was expressed in terms of standard functions and sensitive parameters: α, defined to ensure the observer receives a maximum view of the flame surface and β, the angle of inclination of the differential observer plane. Results from a planar approximation to F are compared with those of PHAST 7.2 simulated results in MATLAB. The values of the planar approximation to F was found to increase with increasing β. This suggests that the larger the value of β, the more the radiation received by a near observer. For $\beta = 48.961^o$, horizontal distance, $X = 30$m, $\alpha = 0.01$, flame length, $L = 12.14$m, tilt angle, $\theta = 55.17^o$, and pool diameter, $D = 5$m: planar and PHAST 7.2 approximations to F were found to coincide up to 6 significant digits and differ by 3.2×10^{-8}. The close agreement between both approximations depend heavily on the choice of flame properties and sensitive parameters. Interestingly, this result may defer for certain choice of sensitive parameter leading to prediction of planar approximation to F way higher or lower than those of PHAST 7.2 approximations. The advantage of this approach is that a knowledge of the plane where the largest concentration of radiation is located will help minimise loss of lives to fire hazards and improve the efficiency of risk safety assessment/management.

Key words— approximation, flame, observer, planar, pool fire, view factor.

1 Introduction

Driving towards a safer, smarter and greener world in the face of emerging industrialisation cannot be achieved without proper safety precautions in the design, construction and operation of liquid hydrocarbon storage facilities, as poor management of accidental releases of liquid hydrocarbons could result in large/small scale pool fire, emitting thermal radiation which could be dangerous to lives and property.

A pool fire whose geometry is assumed cylindrical in shape is the most important type of fire hazard when compared to other fires because of its sheer devastating potential and ability to cause maximum damage in process industries [1]. Its very common in scenarios like: rupture of pipelines transporting fuel and huge volume of fuel stored in tanks. It

*Professor, Department of Mathematical Sciences, University of Bath, Bath, UK. Email: C.J.Budd@bath.ac.uk

†Corresponding author: Department of Mathematical Sciences, New Jersey Institute of Technology (NJIT), USA. Email: uou3@njit.edu

may also arise as a result of leakage or spillage of flammable substances; cracks in vessels or pipelines including abrupt temperature or pressure changes in operating systems [1].

Mannan [2] defined a pool fire as that which occur when flammable liquid spills onto the ground and is ignited. This could be in form of trench fire or fire in a liquid storage tank and may also occur on water whose surface contains the spilled flammable liquid. Additionally, Fay [3] defined pool fire as a diffusion flame driven entirely by gravitational buoyancy. Invariably, a buoyant diffusion flame whose fuel is configured (shaped/formed together) horizontally could be regarded as a pool fire.

Pool fires which could either be modelled as "large pool" or "small pool" depending on the size of the pool diameter, requires careful determination of fire dynamics from evolution time to extinction period. In other words, it involves an in-depth understanding of the nature of combustion, pool spreading, dependence of radiation intensity on flame shape, turbulence and temperature at various flame zones.

It's noteworthy that the dependence of radiation intensity on flame shape and existence of flame zones in effective management of fire hazards, cannot however, be determined without proper conceptualisation of pool fire characteristics: composition of liquid fuel, size and shape of fire, duration of fire, mass burning rate, average flame surface emissive power, atmospheric transmissivity and view factor.

The view factor which of interest in this paper depends on the shape and size of the flame including the position and orientation of the target. In particular, consider an observer at some target position from the flame, the configured proportion of all the radiation that reaches the observer from the flame surface is called the view factor of the flame. Hence, it will not be surprising to anticipate the dependence of view factor on the shape and location of the flame relative to the target.

Several analytic methods have been used to determine the view factor of pool fires [4]. In particular, [5] employed analytic techniques to determine the view factor of tilted cylinders while [6] observed that available methods of analytic solution for predicting view factors of tilted cylinder with circular cross-section and other simple shapes are restricted to certain locations and orientation of the target. View factor prediction method (which divides the flame surface into small parallelogram) [7] for tilted cylinder with target located at ground level, directly under the flame and part of the cylinder surface directly contained within the view field of the target have been scrutinisingly criticised by [6], who observed that, the method made no allowance for the differences in area of these parallelogram as their position change around the circumference of the cylinder.

In order to address these restrictions, [6] developed an area integral for view factor which is evaluated numerically and could cover any geometrical shape by dividing the entire flame surface into triangular area elements which are calculated and summed vectorially provided the triangles have been defined. This numerical method could be used to calculate view factor for modelling radiant heat flux from irregular shapes. Interestingly, its accuracy depends upon the number of triangular mesh (flame elements) since the more the mesh, the more accurate it becomes and the more computational time.

More generally, the view factor between the flame surface and the target (observer location) is calculated over the flame surface that are visible from the location and orientation of the observer. It was derived by application of Lambert's cosine rule [8] and is given by

$$F = \iint_{A_1} \frac{\cos\beta_1 \cos\beta_2}{\pi d^2} dA_1 \tag{1}$$

where,
F is the view factor.
β_1 is the angle between the local normal to the flame element and the line joining this element to target position.
β_2 is the angle between the unit normal specifying the orientation of the elemental target and the line joining the target to the flame element.

d is the distance from the flame surface element to the target element.
dA_1 is the area of flame surface element.
A_1 is the area of the flame surface that can be viewed from the location and orientation of the target.

A restriction to (1) is that any contribution for which $\cos\beta_1$ or $\cos\beta_2$ is negative is neglected. In other words, it is required that $\cos\beta_1 > 0$ and $\cos\beta_2 > 0$ in order to be used in the above calculation.

Our objective in this paper is to determine the expression for the view factor of a 'grain-like' observer near a pool fire whose flame surface 'seen' by the observer is approximately planar. Invariably, we require the observer to be as close as possible to the flame surface so that the flame surface is viewed as a plane and the observer receives a maximum view of the flame. In fact, we would expect that the closer the observer is to flame surface, the more planar the flame surface appears and the higher the view factor (the configured proportion of all the radiation that reaches the observer from the flame surface) of the observer gets. This to a great extent distinguishes our approach in this paper to those found in literature since a better view factor calculation allows for a better risk assessment of the safety of pool fire and how best to minimise the level of radiation damage that can result from it especially for a near observer.

Also of interest is to characterise how the incident angle of radiation affects the configured proportion of all the radiation that reaches the observer from the flame surface. This we intend to address by identifying and analysing some sensitive parameters: α and β.

Our research in this paper could be of interest to DNV GL London UK, since during the *Industrial Day Presentation, March 2015* at University of Bath, UK, they noted that numerically integrating (1) as obtained in PHAST could be computationally expensive. Hence the need to determine if there are analytic solutions.

PHAST (software version 7.2) as will be used in this paper has the capacity to compute F from different fire scenarios by numerically solving (1). One of such will be considered analytically in this paper by planar approximating (1), and subsequently comparing with PHAST 7.2 simulated results for F to determine how accurately they compare. Comparison with PHAST predictions will be considered sufficient for our model validation since according to [9], PHAST calculations have been verified and validated against field measurements from ([10]-[12]). Further details on PHAST 7.2 could be found in section 4.

We have assumed that the pool fire is on land, having no elevation above the ground with horizontal distance away from the observer location. Also, the volume and shape of the cylinder does not change as it tilts to greater angle by greater wind velocity. The observer height is also assumed to be negligible - 'grain-like'.

2 Model Formulation

We formulate the problem for the view factor of a planar flame surface paying attention to the approach presented in Sugden [13]. Though it seem feasible to view the pool fire as cylindrical in shape with curved flame surface, approximating the curved surface as a plane is only feasible if the observer is near the flame surface. Moreover, we require an observer to receive a maximum view of the flame which would only be possible if the observer is directly opposite the tilted pool fire. An observer location near the pool fire would mean an observer only sees a fragment of the flame surface which can realistically be modelled as a plane.

A major advantage of this approach is that it takes into account a fragment of the flame surface where the largest concentration of radiation emanates. The effect of wind ensures the largest configured radiation are concentrated in the plane 'seen' by the observer.

In practice, the merits of this approach is that in the case of fire hazards and risk safety assessments, knowing the wind direction will help avoid/rescue victims of fire hazards from the plane where the configured radiation are concentrated.

There are pockets of limitation to this approach many of which will be mentioned at the end of this paper. However, this approach does not take into account the error that might result in planar approximating the curved flame surface.

Now consider Figure 1 below showing a tilted pool fire assumed to be cylindrical in shape under wind effects.

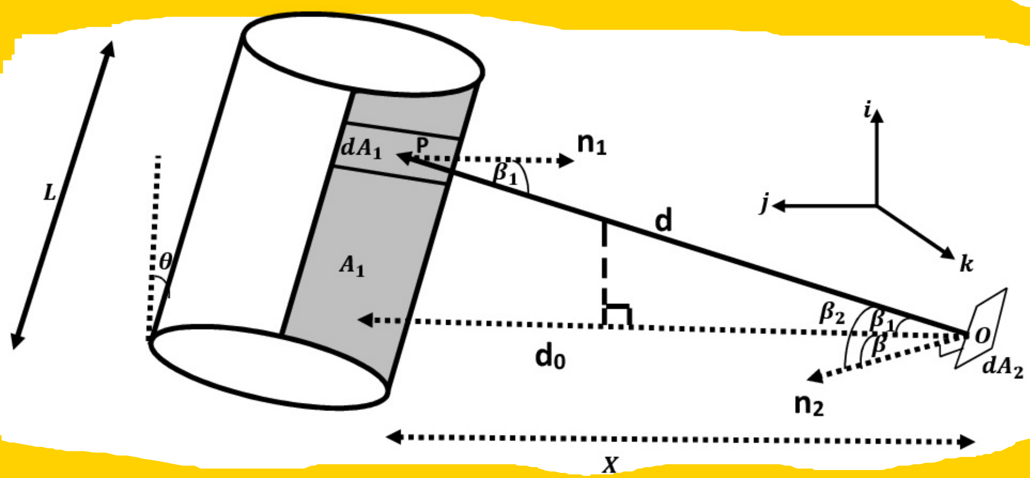

Figure 1: Geometric description of view factor for planar flame surface.

A_1 is the area of the flame seen by the observer, the observer orientation is to receive the maximum view: observer location is directly opposite the wind direction. The normal $\mathbf{d_0}$ to the plane A_1 is the same along A_1, $\mathbf{d}$ is the position vector to an arbitrary point, $P = P(x, y, z)$, in the plane A_1, β_1 is the angle of emission (between the normal, $\mathbf{n_1}$, to the flame element, dA_1, and the position vector, $\mathbf{d_0}$, joining the observer, O, lying in the observer plane dA_2) from P. β_2 is the incidence angle (angle between $\mathbf{d}$ and the normal, $\mathbf{n_2}$ to the observer, O on the observer plane dA_2) of radiation at O from the source plane dA_1. β is an acute angle between the normals $\mathbf{n_1}$ and $\mathbf{n_2}$.

Observe from Figure 1 that the position vector, $\mathbf{d}$, can be written in terms of the right-handed orthonormal system $\{i, j, k\}$ emanating from O so that

$$\mathbf{d} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}. \tag{2}$$

Since $\mathbf{d_0}$ is normal to the plane A_1, we must have that

$$\mathbf{d_0} = d_0\mathbf{j}. \tag{3}$$

Additionally, observe that $y = d_0$ since P lies on A_1 so that (3) gives

$$\mathbf{d} = x\mathbf{i} + d_0\mathbf{j} + z\mathbf{k}. \tag{4}$$

Also, from Figure 1, we see that

$$\mathbf{n_2} = (\sin\beta)\mathbf{i} + (\cos\beta)\mathbf{j}. \tag{5}$$

Now recall that β_2 is the incidence angle of radiation at O from the source plane dA_1. That is, β_2 is the angle between the position vector, $\mathbf{d}$, and the normal, $\mathbf{n_2}$.

Hence, by definition of dot product of two vectors, we have

$$\cos\beta_2 = \frac{x\sin\beta + d_0\cos\beta}{d}. \tag{6}$$

where $d = |\mathbf{d}|$. Also,

$$\cos\beta_1 = \frac{d_0}{d}. \tag{7}$$

Here, we assume d_0 is small relative to d. This is to ensure that the observer receives a maximum view of the flame since the smaller the value of d_0/d, the larger the value of β_1. In practice, this may not always be the case as d could be chosen to be small relative to d_0.

Next, we deduce from Figure 1 and using (1) that the view factor, F, from A_1 to the observer location, O, is given by

$$F = \iint_{A_1} \frac{\cos\beta_1 \cos\beta_2}{\pi d^2} dA_1 \tag{8}$$

where,
$\cos\beta_2 > 0$ such that the angle of incidence at O is acute.
Substituting (6) - (7) into (8) gives

$$F = \iint_{A_1} \frac{\big(x\sin\beta + d_0\cos\beta\big)d_0}{\pi d^4} dA_1 \tag{9}$$

By Fubini's theorem, we can convert the double integral in (9) to an iterated integral given as

$$F = \frac{d_0}{\pi}\int_{x=x_{\min}}^{A}\int_{z=z_{\min}}^{B} \frac{\big(x\sin\beta + d_0\cos\beta\big)}{(x^2+d_0^2+z^2)^2} dz dx \tag{10}$$

Equation (10) represents the problem for planar approximation of the view factor of a tilted flame. It's noteworthy that the limits of integration over the flame surface used in (10) were obtained from the flame coordinates presented in [9].

$$x_{\min} = 0; \quad z_{\min} = 0; \quad A = L\sin\theta; \quad \text{and} \quad B = L\cos\theta \tag{11}$$

where,
θ is the angle of tilt.
L is the flame length.

3 Method of Solutions

In this section, we seek analytic solutions to the formulated view factor problem for the planar approximation of the flame surface.

To evaluate (10), we introduce the following transformations:

$$\xi = \frac{x}{d} \Rightarrow x = \xi d. \tag{12}$$

$$\eta = \frac{z}{d} \Rightarrow z = \eta d. \tag{13}$$

Also, write

$$\alpha = \frac{d_0^2}{2d^2} \Rightarrow d_0 = d\sqrt{2\alpha}. \tag{14}$$

Then

$$F = \frac{d_0}{\pi}\int_{\xi=x_{\min}/d}^{A/d}\int_{\eta=z_{\min/d}}^{B/d} \frac{\xi d\sin\beta + d_0\cos\beta}{(\xi^2 d^2 + d_0^2 + \eta^2 d^2)^2} d^2 d\eta d\xi. \tag{15}$$

Using (14), we note from (15) that

$$\frac{\xi d\sin\beta + d_0\cos\beta}{(\xi^2 d^2 + d_0^2 + \eta^2 d^2)^2} d^2 = \frac{\xi\sin\beta}{d(2\alpha+\xi^2+\eta^2)^2} + \frac{d_0\cos\beta}{d^2(2\alpha+\xi^2+\eta^2)^2}. \tag{16}$$

Now, multiply (15) by $\frac{\pi}{d_0}$ and substitute (16) into (15) to get

$$\frac{\pi}{d_0}F = \frac{\sin\beta}{d}\int_{\xi=x_{\min}/d}^{A/d}\xi d\xi\int_{\eta=z_{\min/d}}^{B/d}\frac{d\eta}{(2\alpha+\xi^2+\eta^2)^2} + \frac{d_0\cos\beta}{d^2}\int_{\xi=x_{\min}/d}^{A/d}d\xi\int_{\eta=z_{\min}/d}^{B/d}\frac{d\eta}{(2\alpha+\xi^2+\eta^2)^2}d\eta. \tag{17}$$

Further multiplying (17) by d and imposing (14) gives

$$\frac{\pi}{\sqrt{2\alpha}}F = \sin\beta\int_{\xi=x_{\min}/d}^{A/d}\xi d\xi\int_{\eta=z_{\min}/d}^{B/d}\frac{d\eta}{(2\alpha+\xi^2+\eta^2)^2} + \sqrt{2\alpha}\cos\beta\int_{\xi=x_{\min}/d}^{A/d}d\xi\int_{\eta=z_{\min}/d}^{B/d}\frac{d\eta}{(2\alpha+\xi^2+\eta^2)^2}. \tag{18}$$

Next, define:

$$J(\gamma) = \int_{\eta=z_{\min}/d}^{B/d}\frac{d\eta}{(\gamma^2+\eta^2)^2} \tag{19}$$

so that (18) becomes

$$\frac{\pi}{\sqrt{2\alpha}}F = \sin\beta\int_{\xi=x_{\min}/d}^{A/d}\xi J(\sqrt{2\alpha+\xi^2})d\xi + \sqrt{2\alpha}\cos\beta\int_{\xi=x_{\min}/d}^{A/d}J(\sqrt{2\alpha+\xi^2})d\xi. \tag{20}$$

Set $\eta = \gamma z$ and substitute into (19) to obtain

$$J(\gamma) = \int_{\eta=z_{\min}/\gamma d}^{B/\gamma d}\frac{\gamma dz}{\gamma^4(1+z^2)^2} = \frac{1}{\gamma^3}\int_{\eta=z_{\min}/\gamma d}^{B/\gamma d}\frac{dz}{(1+z^2)^2}$$
$$= \frac{1}{2\gamma^3}\left[\left\{\frac{B/\gamma d}{1+(B/\gamma d)^2} - \frac{z_{\min}/\gamma d}{1+(z_{\min}/\gamma d)^2}\right\} + \left\{\arctan(B/\gamma d) - \arctan(z_{\min}/\gamma d)\right\}\right] \tag{21}$$

since $\int\frac{dz}{(1+z^2)^2} = \frac{z}{2(1+z^2)} + \frac{1}{2}\arctan(z) + c$. In other words,

$$J(\gamma) = \frac{G}{2\gamma^3} \Rightarrow J(\sqrt{2\alpha+\xi^2}) = \frac{G}{4\sqrt{2}}\frac{1}{(\alpha+\xi^2/2)^{3/2}} \tag{22}$$

where

$$G = \frac{B/\gamma d}{1+(B/\gamma d)^2} - \frac{z_{\min}/\gamma d}{1+(z_{\min}/\gamma d)^2} + \arctan(B/\gamma d) - \arctan(z_{\min}/\gamma d). \tag{23}$$

Consequently, if we write

$$S = \int_{\xi=x_{\min}/d}^{A/d}\xi J(\sqrt{2\alpha+\xi^2})d\xi = \frac{G}{4\sqrt{2}}\int_{\xi=x_{\min}/d}^{A/d}\frac{\xi d\xi}{(\alpha+\xi^2/2)^{3/2}} \tag{24}$$

$$R = \int_{\xi=x_{\min}/d}^{A/d}J(\sqrt{2\alpha+\xi^2})d\xi = \frac{G}{4\sqrt{2}}\int_{\xi=x_{\min}/d}^{A/d}\frac{d\xi}{(\alpha+\xi^2/2)^{3/2}} \tag{25}$$

then (20) can compactly be written as:

$$\frac{\pi}{\sqrt{2\alpha}}F = S\sin\beta + \sqrt{2\alpha}R\cos\beta \tag{26}$$

which simplifies to

$$F = \left(\frac{\sqrt{2\alpha}}{\pi}S\right)\sin\beta + \left(\frac{2\alpha}{\pi}R\right)\cos\beta. \tag{27}$$

Equation (27) gives the planar approximation to the view factor of a tilted pool fire.

3.1 Estimation of S

Let $u = \alpha + \xi^2/2$. Then, $d\xi = du/\xi$, which upon substitution into (24) gives

$$\begin{aligned} S &= \frac{G}{4\sqrt{2}} \int_{u=\alpha+x_{\min}^2/2d^2}^{\alpha+A^2/2d^2} \frac{du}{u^{3/2}} \\ &= \frac{G}{2\sqrt{2}} \left[-\frac{1}{\sqrt{u}} \right] \Bigg|_{\alpha+x_{\min}^2/2d^2}^{\alpha+A^2/2d^2} = \frac{G}{2\sqrt{2}} \left[\frac{1}{\sqrt{\alpha + x_{\min}^2/2d^2}} - \frac{1}{\sqrt{\alpha + A^2/2d^2}} \right]. \end{aligned} \tag{28}$$

3.2 Estimation of R

Let $\xi = -\sqrt{2\alpha}\cot\phi$, $d\xi = -\sqrt{2\alpha}\csc^2\phi d\phi$. Then upon substitution into (25) gives

$$\begin{aligned} R &= -\frac{G}{4\sqrt{2}} \int_{\phi=\cot^{-1}(-\frac{x_{\min}}{d\sqrt{2\alpha}})}^{\cot^{-1}(-\frac{A}{d\sqrt{2\alpha}})} \frac{\sqrt{2\alpha}\csc^2\phi d\phi}{\alpha^{3/2}(1+\cot^2\phi)^{3/2}} \\ &= -\frac{G}{4\sqrt{\alpha}} \int_{\phi=\cot^{-1}(-\frac{x_{\min}}{d\sqrt{2\alpha}})}^{\cot^{-1}(-\frac{A}{d\sqrt{2\alpha}})} \frac{d\phi}{\csc\phi} = -\frac{G}{4\sqrt{\alpha}} \int_{\phi=\cot^{-1}(-\frac{x_{\min}}{d\sqrt{2\alpha}})}^{\cot^{-1}(-\frac{A}{d\sqrt{2\alpha}})} \sin\phi d\phi \\ &= \frac{G}{4\sqrt{\alpha}} \left[\cos\left(\cot^{-1}\left(-\frac{A}{d\sqrt{2\alpha}}\right)\right) - \cos\left(\cot^{-1}\left(-\frac{x_{\min}}{d\sqrt{2\alpha}}\right)\right) \right]. \end{aligned} \tag{29}$$

4 Simulation of Results

In this section we present a detailed discussion of our results aimed at drawing sensible conclusions to our model. Recall that our main objective is to seek a planar approximation to F via standard functions for a 'grain-like' observer near a tilted pool fire. This we have achieved in section 3 and is given by (27). Also, (27) expresses the view factor in terms of two sensitive parameters: α and β. However, how sensitive these parameters are, we hope to see in a sequel.

Now it remains to characterise how the incident angle of radiation affects the configured proportion of all the radiation that reaches the observer from the flame surface.

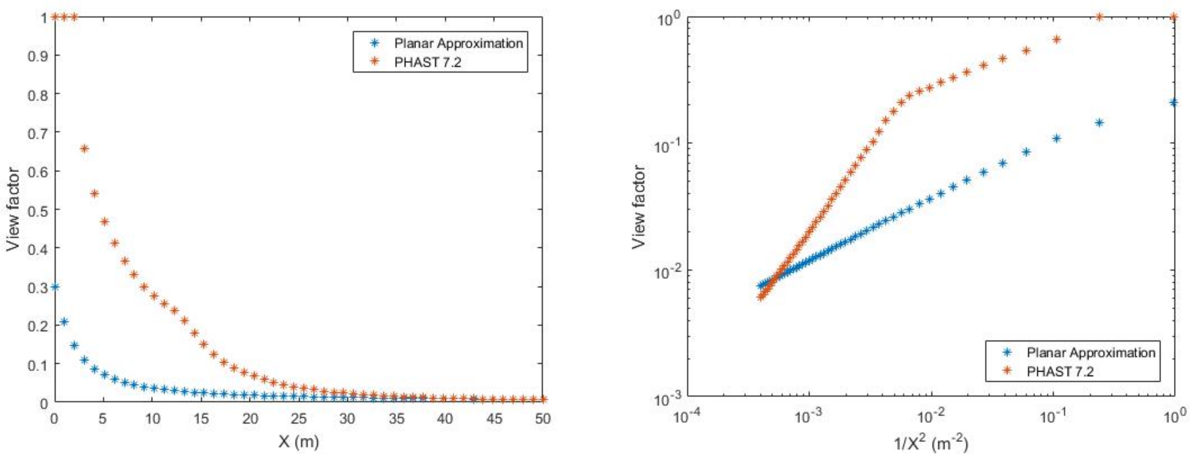

Figure 2: Comparison of various approximations to F, with $X = 50$m, $\beta = 85^o$, $\alpha = 0.01$, $\gamma = 0.5$, $L = 12.14$m, $\theta = 55.17^o$, $D = 5$m.

To do this, simulated data for hydrocarbon fuel material: N-HEPTANE have been generated from PHAST (software version 7.2). Here, any choice of fuel material could be used. However, we note that N-HEPTANE combusts as a sooty fire and could be regarded as an inhomogeneous flame medium. PHAST 7.2 uses the Exposure Model which calculates: whether an observer is engulfed by a flame and the geometric view factor for an observer exposed to that flame. It also provides the facility for maximising the view factor by varying the observer orientation [14].

View factor calculation from PHAST 7.2 (based on correlated data from ([10], [15]) takes inputs (could be user-specified) and outputs quantities such as view factor, F, flame length, L, and flame angle, θ. Outputs such as L and θ are required for the calculation of planar view factor which are compared in MATLAB with those of PHAST 7.2 calculations.

Additionally, comparison between the computational times (run times) for both approximations will not be possible at this stage since PHAST 7.2 does not provide computational times for its view factor calculations. It however provides an exposure time for which an observer is exposed to the flame. Here, our simulation of results is based on an exposure time of 20 seconds.

Now, observe from Figures (2 - 3) that F decreases with increasing observer distance from the pool fire. This is physically reasonable since the farther the observer is from the pool fire, the lesser the view over the flame surface and the lesser the radiation received.

Evident from Figure 3 is the fact that for an observer $X = 200$m away from a pool fire of 5m in diameter, PHAST 7.2 results gave a view factor higher than the planar approximation. This suggests that planar approximation may not be suitable for observer distances from $X = 200$m away from the pool fire.

Apparent from Figure 2 is the fact that the planar approximation gave values of F higher than PHAST 7.2 approximations for $X = 50$m away from the pool fire. One may argue from the *loglog* plot of Figure 2 that both approximations are likely to agree closely for near observer locations. Hence, a necessary and sufficient open question would be:

What range of values of X could be regarded as small distances away from the pool?

This has become important since a clear knowledge of the limits of X for near observer, would help, easily determine how suitable the planar approximation could be used in a typical pool fire scenario.

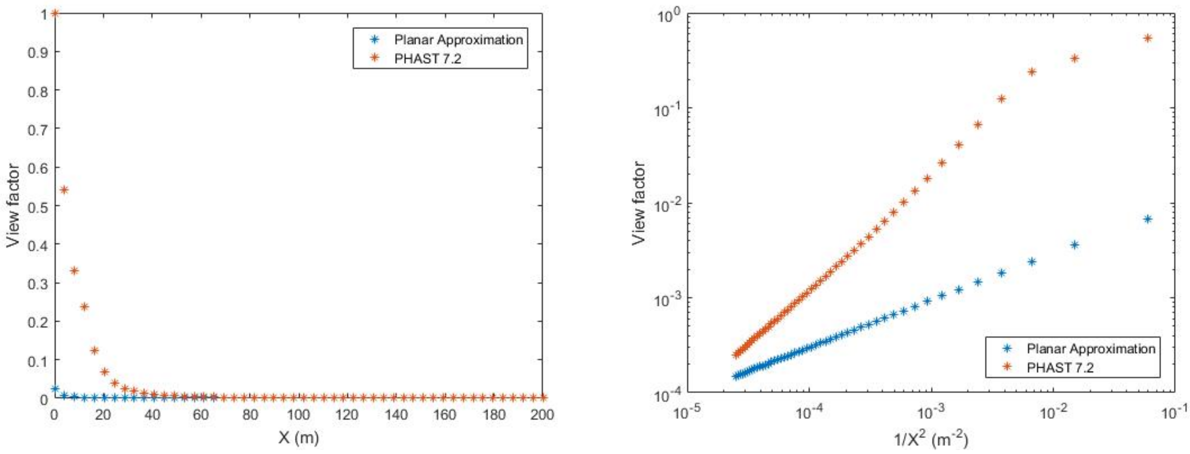

Figure 3: Comparison of various approximations to F, with $X = 200$m, $\beta = 85^o$, $\alpha = 0.01$, $\gamma = 0.5$, $L = 12.14$m, $\theta = 55.17^o$, $D = 5$m.

Also of interest is the fact that PHAST 7.2 predicts view factor of 1 for an observer at both the center of the pool and at certain distances within the flame from the pool center up to the flame surface, which however, decreases farther away as corroborated in Figures (2 - 3). This suggests that there will be no significant difference in calculating the view factor of a target near observer location of interest from the center of the pool fire or from the flame surface. A consequence of the above observations is that both planar approximation and PHAST 7.2 could predict view factor for an observer at the center of the pool. Also, we note that the target positions of interest (being calculated) affects how much of the view of an observer at different locations within the flame.

Consistently, though all the view factor approximations (planar and PHAST 7.2) shown in Figures (2 - 3) decreases (decays polynomially) as the observer distance increases away from the flame surface, a major limitation is the difficulty in determining the rate of decay.

Another major distinction between Figures (2 - 3) is the presence of 'hump' when the flame length is approximately equal to some observer location. This phenomenon is very prevalent in PHAST 7.2 calculation of view factor for near observer.

Interestingly, this change in decay rate would be consistent with the planar approximation being good for near observer location whereas, a point or line for far afield observer location. In particular, a quick look at Figure 3 for $X = 200$m shows no change in decay rate.

Next, we consider the sensitivity analysis of the following parameters: α and β found in (27). Also of interest is to investigate the assertions established from Figures (2 - 3) that the planar approximation would likely agree closely with PHAST 7.2 predictions for near targets from the pool fire.

4.1 Sensitivity Analysis for $\alpha = \frac{d_0^2}{2d^2}$

We investigate the sensitivity of α using $X = 30$m, $\beta = 48.961^o$, $\gamma = 0.77$, $L = 12.14$m, $\theta = 55.17^o, D = 5$m.

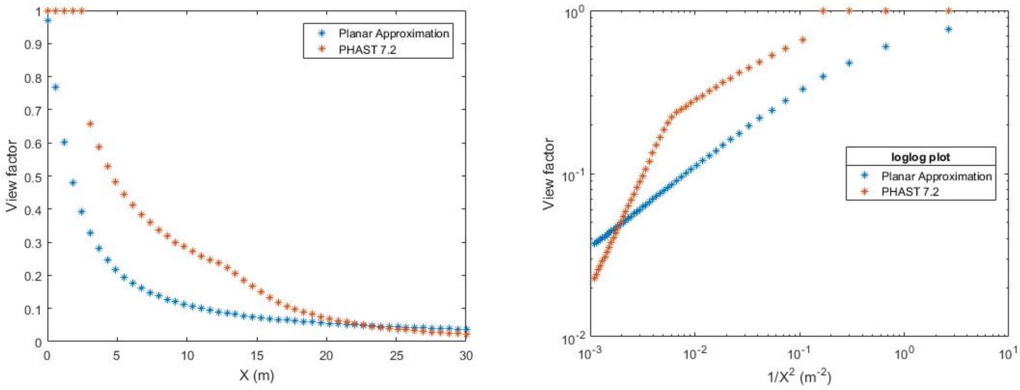

Figure 4: Comparison of various approximations to view factor, F, with $\beta = 48.961^o, X = 30$m, $\alpha = 0.02, \gamma = 0.77, L = 12.14$m, $\theta = 55.17^o, D = 5$m.

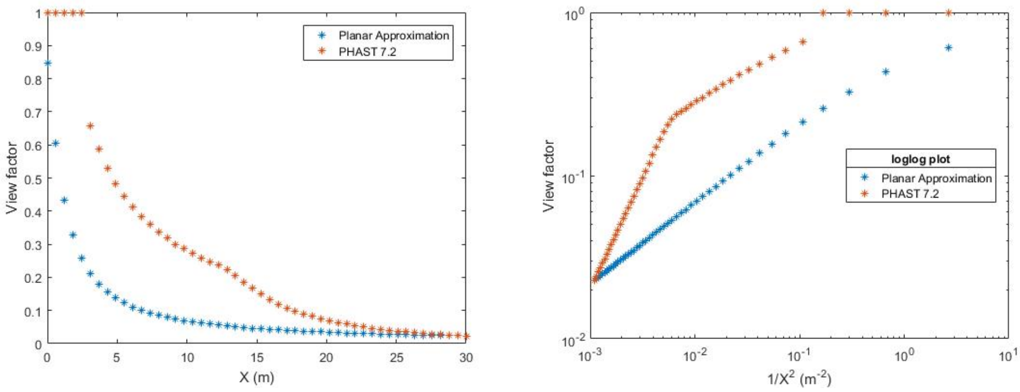

Figure 5: Comparison of various approximations to view factor, F, with $\beta = 48.961^o, X = 30$m, $\alpha = 0.01, \gamma = 0.77, L = 12.14$m, $\theta = 55.17^o, D = 5$m.

Recall that $\alpha = d_0^2/2d^2$, where the characterisation of d and d_0 follows from Figure 1. Therein, we required that α is sensibly as small as possible (d_0 small relative to d).

Hence, the need to investigate the sensitivity of α used, to determine how small it could be for close agreement between planar and PHAST 7.2 view factor calculations.

Now, observe from Figures (4 - 6) that though we require α to be sensibly small for a very close agreement between planar and PHAST 7.2 approximations, caution must be taken on the choice of α. We suspect that very small values would tend to give view factor way lower/higher than PHAST 7.2 predictions.

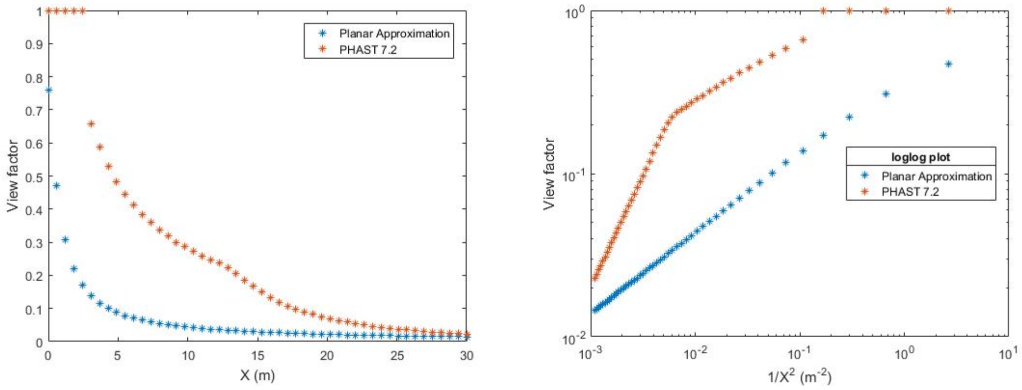

Figure 6: Comparison of various approximations to view factor, F, with $\beta = 48.961^o, X = 30$m, $\alpha = 0.005, \gamma = 0.77, L = 12.14$m, $\theta = 55.17^o, D = 5$m.

More generally, for relatively small horizontal distances, X, away from the flame, the planar and PHAST 7.2 approximations would agree closely provided that α is sensibly small. Moreover, we would anticipate that a sensible choice for α would be: $0.01 - \epsilon \leq \alpha \leq 0.01$.

4.2 Sensitivity Analysis for β

Here, we consider $X = 30$m, $\alpha = 0.01, \gamma = 0.77, L = 12.14$m, $\theta = 55.17^o, D = 5$m for the sensitivity analysis of β.

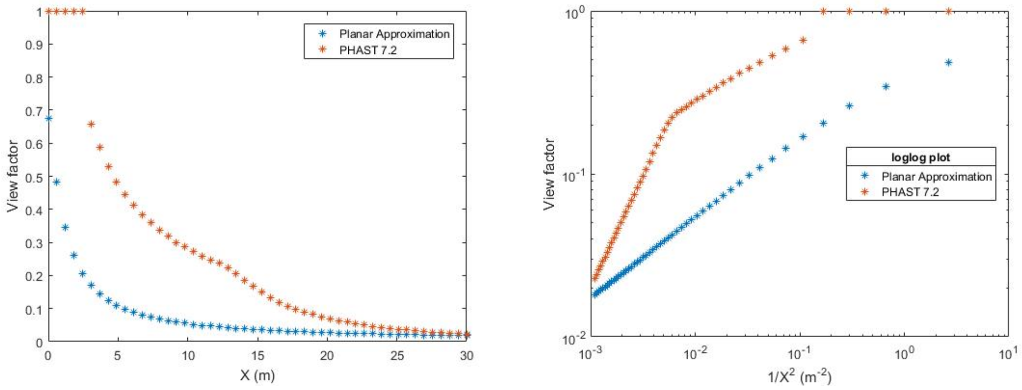

Figure 7: Comparison of various approximations to view factor, F, with $\beta = 20^o$, $\alpha = 0.01, \gamma = 0.77$, $X = 30$m, $L = 12.14$m, $\theta = 55.17^0$, $D = 5$m.

The sensitivity analysis shown in Figures (7 - 10) and corroborated in Table 1 (Appendix A) suggests that, the larger the values of β, the closer the view factor predictions of PHAST 7.2 and planar approximations. In other words, the choice of β would be as sensibly large as possible for close agreement with PHAST 7.2 predictions. Physically, β large enough would guarantee the observer has a maximum view factor of the flame and hence maximum radiation.

In summary, we have been able to establish from Figures (7 - 10) and Table 1 (Appendix A) that depending on the values of β, the planar approximation to F could either

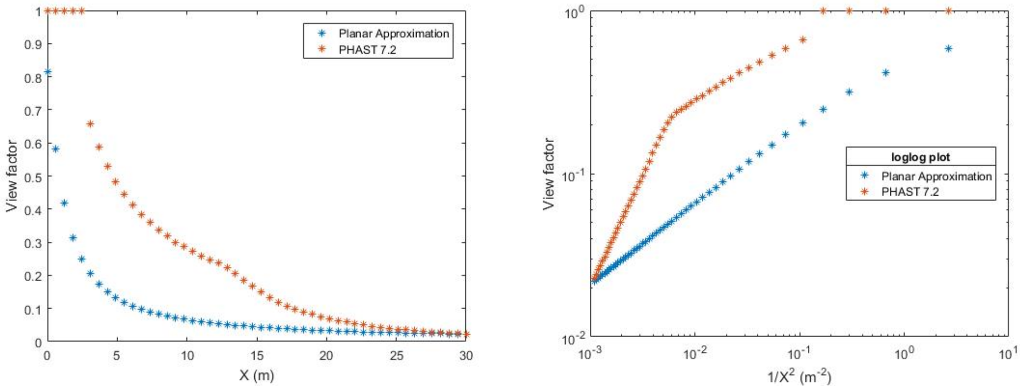

Figure 8: Comparison of various approximations to view factor, F, with $\beta = 40^o$, $\alpha = 0.01, \gamma = 0.77$, $X = 30$m, $L = 12.14$m, $\theta = 55.17^o$, $D = 5$m.

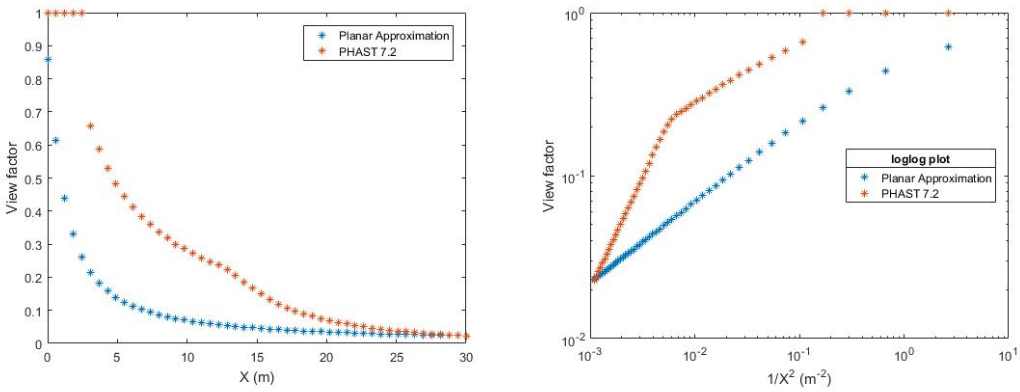

Figure 9: Comparison of various approximations to view factor, F, with $\beta = 60^o$, $\alpha = 0.01, \gamma = 0.77$, $X = 30$m, $L = 12.14$m, $\theta = 55.17^o$, $D = 5$m.

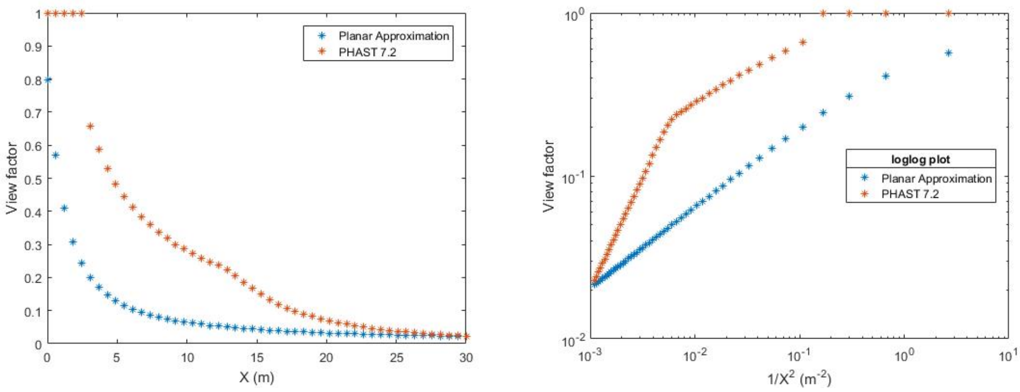

Figure 10: Comparison of various approximations to view factor, F, with $\beta = 80^o$, $\alpha = 0.01, \gamma = 0.77$, $X = 30$m, $L = 12.14$m, $\theta = 55.17^o$, $D = 5$m.

be lower or higher than those of PHAST 7.2 predictions. Additionally, both approximations have been found to coincide upto 6 significant digits for $\beta = 48.961^o, \gamma = 0.77$ and $\alpha = 0.01$ and differ by 3.2×10^{-8}.

5 Conclusions

We have been able to formulate, solve and calculate the view factor of a tilted pool fire by taking a fragment of the flame surface with the largest concentration of the configured radiation concentrated due to wind effects in the plane 'seen' by an observer located opposite the wind direction. Here, a knowledge of the wind direction will help avoid/rescue victims of fire hazards from the plane where the configured radiation is located. This we strongly believe will help minimise loss of lives and property to fire hazards and improve the efficiency of risk safety assessment/management.

A very important assumption is that the orientation of the observer is so as to receive the maximum view. Thus, the view factor integration is restricted to the area 'seen' by the observer. This could further be investigated by considering a problem for multiple observer around the pool fire since they would not always have an orientation so as to receive a maximum view. It's noteworthy that the orientation of the observer also affects the view factor calculations which would not always be the same for all observer around the pool.

The planar approximation to the view factor was found to be very flexible: could predict view factors higher or lower than PHAST 7.2 results depending on the choice of sensitive parameters. More interestingly, it could coincide with PHAST 7.2 predictions.

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Appendix A.

$F_{\text{PHAST 7.2}} = 0.02288$	
β^o	F_{planar}
5	0.01387
10	0.01544
15	0.01689
20	0.01821
25	0.01939
30	0.02042
35	0.02130
40	0.02202
45	0.02257
50	0.02295
55	0.02315
60	0.02318
65	0.02303
70	0.02271
75	0.02221
80	0.02154
85	0.02071

Table 1: Illustrates the relationship between the β and the planar view factor with $\alpha = 0.01, \gamma = 0.77, L = 12.14\text{m}, \theta = 55.17^0, D = 5\text{m}$.